EXERCISE 12.1

1. Evaluate:

(i) 3^{-2} (ii) $(-4)^{-2}$ (iii) $(1/2)^{-5}$

Solution:

(i) $3^{-2} = (1/3)^2$

$$\left[\because\ a^{-m} = \frac{1}{a^m}\right]$$

$= 1/9$

(ii) $(-4)^{-2} = (1/-4)^2$

$$\left[\because\ a^{-m} = \frac{1}{a^m}\right]$$

$= 1/16$

(iii) $(1/2)^{-5} = (2/1)^5$

$$\left[\because\ a^{-m} = \frac{1}{a^m}\right]$$

$= 2^5$

$= 32$

2. Simplify and express the result in power notation with a positive exponent:

(i) $(-4)^4 \div (-4)^8$

(ii) $(1/2^3)^2$

(iii) $-(3)^4 \times (5/3)^4$

(iv) $(3^{-7} \div 3^{-10}) \times 3^{-5}$

(v) $2^{-3} \times (-7)^{-3}$

Solution:

(i)

$$\left(-4\right)^5 \div \left(-4\right)^8$$

$= (-4)^5/(-4)^8$

$$\left[\because \quad a^m \div a^n = a^{m-n}\right]$$

$= (-4)^{5-8}$

$= 1/(-4)^3$

(ii) $(1/2^3)^2$

$= 1^2/(2^3)^2$

$$\left[\because \quad \left(\frac{a}{b}\right)^m = \frac{a^m}{a^n}\right]$$

$= 1/2^{3\times2} = 1/2^6$

$$\left[\because \quad \left(a^m\right)^n = a^{m\times n}\right]$$

(iii) $-(3)^4\times(5/3)^4$

$$(-3)^4 \times \left(\frac{5}{3}\right)^4 = (-3)^4 \times \frac{5^4}{3^4}$$

$$\left[\because \quad \left(\frac{a}{b}\right)^m = \frac{a^m}{a^n}\right]$$

$= (-1)^4\times3^4\times(5^4/3^4)$

$$\left[\because \quad \left(ab\right)^m = a^m b^m\right] =$$

$3^{(4-4)}\times5^4$

$$\left[\because \quad a^m \div a^n = a^{m-n}\right]$$

$= 3^0\times5^4 = 5^4$

$$\left[\because \quad a^0 = 1\right]$$

(iv) $\left(3^{-7} \div 3^{-10}\right)\times3^{-5}$

$= (3^{-7}/3^{-10})\times 3^{-5}$

$= 3^{-7-(-10)} \times 3^{-5}$

$$\left[\because\ a^m \div a^n = a^{m-n}\right]$$

$= 3_{(-7+10)} \times 3_{-5}$

$= 3^3 \times 3^{-5}$

$= 3_{(3+-5)}$

$$\left[\because\ a^m \times a^n = a^{m+n}\right]$$

$= 3^{-2}$

$= 1/3^2$

$$\left[\because\ a^{-m} = \frac{1}{a^m}\right]$$ **(v)**

$2_{-3} \times (-7)_{-3}$

$= (2 \times -7)^{-3}$

(Because $a^m \times b^m = (ab)^m$)

$= 1/(2 \times -7)^3$

$$\left[\because\ a^{-m} = \frac{1}{a^m}\right]$$

$= 1/(-14)^3$

3. Find the value of:

(i) $(3^0 + 4^{-1}) \times 2^2$

(ii) $(2_{-1} \times 4_{-1}) \div 2_{-2}$

(iii) $(1/2)^{-2} + (1/3)^{-2} + (1/4)^{-2}$

(iv) $(3_{-1} + 4_{-1} + 5_{-1})_0$

(v) $\{(-2/3)^{-2}\}^2$

Solution:

(i) $(3^0 + 4^{-1}) \times 2^2 = (1 + (1/4)) \times 2^2$

$$\left[\because\ a^{-m} = \frac{1}{a^m}\right]$$

$= ((4+1)/4)\times 2^2$

$= (5/4)\times 2^2$

$= (5/2^2)\times 2^2$

$= 5\times 2_{(2-2)}$

$\left[\because\ a^m \div a^n = a^{m-n}\right]$

$= 5\times 2^0$

$= 5\times 1 = 5$

$\left[\because\ a^{-m} = \dfrac{1}{a^m}\right]$ **(ii)$(2^{-1}\times 4^{-1})\div 2^{-2}$**

$= [(1/2)\times(1/4)] \div (1/4)$

$\left[\because\ a^{-m} = \dfrac{1}{a^m}\right]$

$= (1/2\times 1/2^2)\div 1/4$

$= 1/2^3 \div 1/4$

$= (1/8)\times(4)$

$= 1/2$

(iii) $(1/2)^{-2}+(1/3)^{-2}+(1/4)^{-2}$

$= (2_{-1})_{-2}+(3_{-1})_{-2}+(4_{-1})_{-2}$

$\left[\because\ a^{-m} = \dfrac{1}{a^m}\right]$

$= 2_{(-1\times -2)}+3_{(-1\times -2)}+4_{(-1\times -2)}$

$\left[\because\ \left(a^m\right)^n = a^{m\times n}\right]$

$= 2^2+3^2+4^2$

$= 4+9+16$

$=29$

(iv) $(3^{-1}+4^{-1}+5^{-1})^0$

$= 1$

$$\left[\because \quad a^0 = 1\right]$$

(v) $\{(-2/3)^{-2}\}^2 = (-2/3)^{-2\times2}$

$$\left[\because \quad \left(a^m\right)^n = a^{m\times n}\right]$$

$= (-2/3)^{-4}$

$= (-3/2)^4$

$$\left[\because \quad a^{-m} = \frac{1}{a^m}\right]$$

$= 81/16$

4. Evaluate:

(i) $(8^{-1}\times5^3)/2^{-4}$

(ii) $(5^{-1}\times2^{-2})\times6^{-1}$

Solution:

(i) $(8^{-1}\times5^3)/2^{-4}$

$$\frac{8^{-1}\times5^3}{2^{-4}} = \frac{\left(2^3\right)^{-1}\times5^3}{2^{-4}} = \frac{2^{-3}\times5^3}{2^{-4}}$$

$$\left[\because \quad \left(a^m\right)^n = a^{m\times n}\right]$$

$=$

$$2^{-3-(-4)}\times5^3 = 2^{-3+4}\times5^3$$

$$\left[\because \quad a^m \div a^n = a^{m-n}\right]$$

$= 2\times125 = 250$

(ii) $(5^{-1}\times2^{-2})\times6^{-1}$

$$\left(5^{-1} \times 2^{-1}\right) \times 6^{-1} = \left(\frac{1}{5} \times \frac{1}{2}\right) \times \frac{1}{6}$$

$$\left[\because \quad a^{-m} = \frac{1}{a^m}\right]$$

$= (1/10) \times 1/6$

$= 1/60$

5. Find the value of *m* for which $5^m \div 5^{-3} = 5^5$

Solution: 5^m

$\div 5^{-3} = 5^5$

$5_{(m-(-3))} = 5_5$

$$\left[\because \quad a^m \div a^n = a^{m-n}\right]$$

$5_{m+3} = 5_5$

Comparing exponents on both sides, we get

m+3 = 5 m = 5-3 m = 2

6. Evaluate:

(i)

$$\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}$$

(ii)

$$\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4}$$

Solution:

(i)

$$\left\{\left(\frac{1}{3}\right)^{-1}-\left(\frac{1}{4}\right)^{-1}\right\}=\left\{\left(\frac{3}{1}\right)^{1}-\left(\frac{4}{1}\right)^{1}\right\}$$

$$\left[\because\ a^{-m}=\frac{1}{a^{m}}\right]$$

= 3-4

= -1

(ii)

$$\left(\frac{5}{8}\right)^{-7}\times\left(\frac{8}{5}\right)^{-4}=\frac{5^{-7}}{8^{-7}}\times\frac{8^{-4}}{5^{-4}}$$

$$\left[\because\ \left(\frac{a}{b}\right)^{m}=\frac{a^{m}}{b^{m}}\right]$$

=

$$5^{-7-(-4)}\times8^{-4-(-7)}$$

$$\left[\because\ a^{m}\div a^{n}=a^{m-n}\right]$$

=

$$5^{-7+4}\times8^{-4+7}$$

$$=5^{-3}\times8^{3}=$$

$$\frac{8^{3}}{5^{3}}$$

$$\left[\because\ a^{-m}=\frac{1}{a^{m}}\right]$$

= 512/125

7. Simplify the following:

(i)

$$\frac{25\times t^{-4}}{5^{-3}\times10\times t^{-8}}\quad(t\neq0)$$

(ii)

$$\frac{3^{-5}\times10^{-5}\times125}{5^{-7}\times6^{-5}}$$

Solution:

(i)

$$\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}}$$

$$= \frac{5^2 \times t^{-4}}{5^{-3} \times 5 \times 2 \times t^{-8}}$$

$$= \frac{5^{2-(-3)-1} \times t^{-4-(-8)}}{2}$$

$$\left[\because \ a^m \div a^n = a^{m-n}\right]$$

$$=$$

$$\frac{5^{2+3-1} \times t^{-4+8}}{2} = \frac{5^4 \times t^4}{2}$$

$$\frac{625}{2} t^4$$

$$=$$

(ii)

$$\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$$

$$= \frac{3^{-5} \times (2 \times 5)^{-5} \times 5^3}{5^{-7} \times (2 \times 3)^{-5}}$$

$$= \frac{3^{-5} \times 2^{-5} \times 5^{-5} \times 5^3}{5^{-7} \times 2^{-5} \times 3^{-5}}$$

$$\left[\because \ (ab)^m = a^m b^m\right]$$

$$=$$

$$\frac{3^{-5} \times 2^{-5} \times 5^{-5+3}}{5^{-7} \times 2^{-5} \times 3^{-5}} = \frac{3^{-5} \times 2^{-5} \times 5^{-2}}{5^{-7} \times 2^{-5} \times 3^{-5}}$$

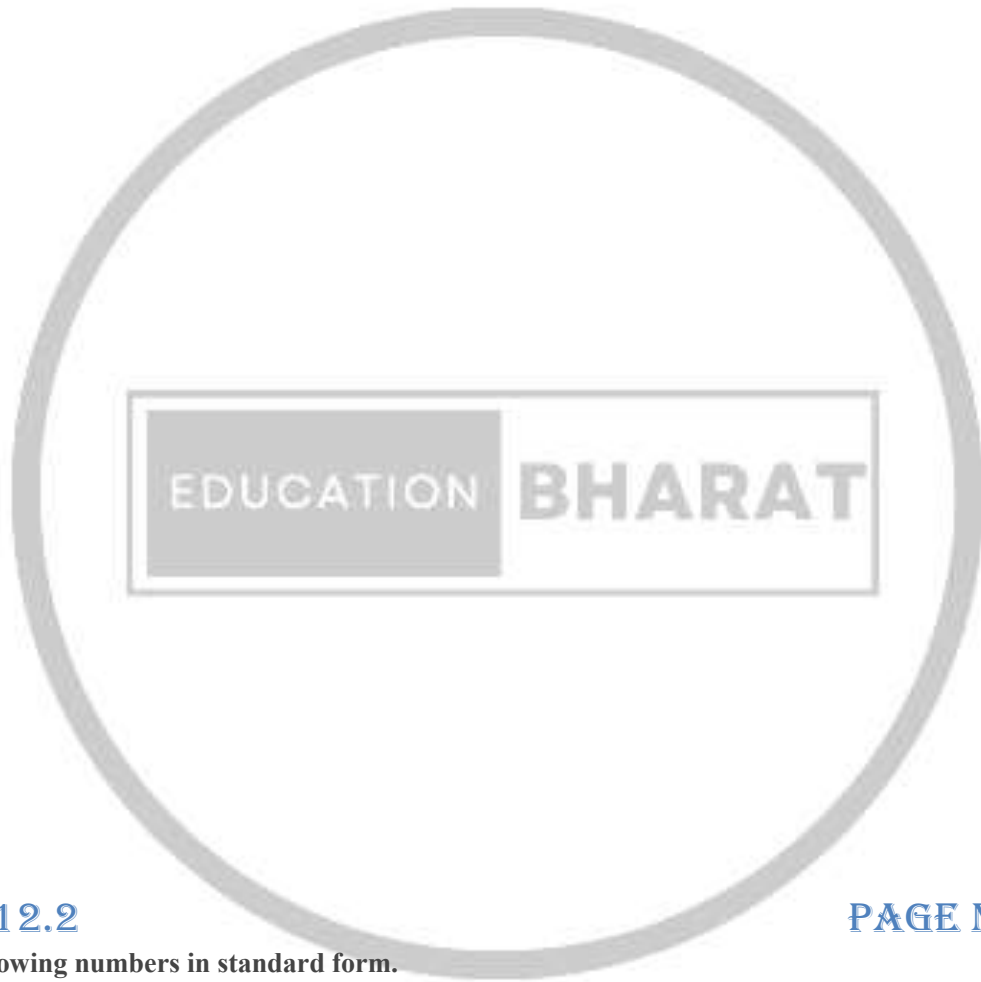$$\left[\because \ a^m \times a^n = a^{m+n}\right]$$

$$=$$

$$3^{-5-(-5)} \times 2^{-5-(-5)} \times 5^{-2-(-7)}$$

$$\left[\because \ a^m \div a^n = a^{m-n}\right]$$

$$=$$

$3^{-5+5} \times 2^{-5+5} \times 5^{-2+7}$

$3^0 \times 2^0 \times 5^5$ =

$= 1 \times 1 \times 3125$

$\left[\because \quad a^0 = 1 \right]$

$= 3125$

EXERCISE 12.2

1. Express the following numbers in standard form.

(i) 0.0000000000085

(ii) 0.00000000000942

(iii) 6020000000000000

(iv) 0.00000000837

(v) 31860000000

Solution:

(i) $0.0000000000085 = 0.0000000000085 \times (10^{12}/10^{12}) = 8.5 \times 10^{-12}$

(ii) $0.00000000000942 = 0.00000000000942 \times (10^{12}/10^{12}) = 9.42 \times 10^{-12}$

(iii) $6020000000000000 = 6020000000000000 \times (10^{15}/10^{15}) = 6.02 \times 10^{15}$

(iv) $0.00000000837 = 0.00000000837 \times (10^{9}/10^{9}) = 8.37 \times 10^{-9}$

(v) $31860000000 = 31860000000 \times (10^{10}/10^{10}) = 3.186 \times 10^{10}$

2. Express the following numbers in the usual form.

(i) 3.02×10^{-6}

(ii) 4.5×10^{4}

(iii) 3×10^{-8}

(iv) 1.0001×10^{9}

(v) 5.8×10^{12} (vi) 3.61492×10^{6}

Solution:

(i) $3.02 \times 10^{-6} = 3.02/10^{6} = 0.00000302$

(ii) $4.5 \times 10^{4} = 4.5 \times 10000 = 45000$

(iii) $3 \times 10^{-8} = 3/10^{8} = 0.00000003$

(iv) $1.0001 \times 10^{9} = 1000100000$

(v) $5.8 \times 10^{12} = 5.8 \times 1000000000000 = 5800000000000$

(vi) $3.61492 \times 10^{6} = 3.61492 \times 1000000 = 3614920$

3. Express the number appearing in the following statements in standard form.

(i)   1 micron is equal to 1/1000000 m.
(ii) Charge of an electron is 0.000, 000, 000, 000, 000, 000, 16 coulomb.
(iii) Size of bacteria is 0.0000005 m
(iv) Size of a plant cell is 0.00001275 m (v) Thickness of a thick paper is 0.07 mm

 Solution:

(i)   1 micron = 1/1000000

$= 1/10^{6}$

$= 1 \times 10^{-6}$

(ii) Charge of an electron is 0.00000000000000000016 coulombs

$= 0.00000000000000000016 \times 10^{19}/10^{19}$

$= 1.6 \times 10^{-19}$ coulomb

(iii) Size of bacteria = 0.0000005

$= 5/10000000 = 5/10^7 = 5 \times 10^{-7}$ m

(iv) Size of a plant cell is 0.00001275 m

$= 0.00001275 \times 10^5/10^5$

$= 1.275 \times 10^{-5}$ m

(v) Thickness of a thick paper = 0.07 mm

0.07 mm $= 7/100$ mm $= 7/10^2 = 7 \times 10^{-2}$ mm

4. In a stack, there are 5 books, each having a thickness of 20 mm and 5 paper sheets, each having a thickness of 0.016 mm. What is the total thickness of the stack?

Solution:

Thickness of one book = 20 mm

Thickness of 5 books = 20×5 = 100 mm

Thickness of one paper = 0.016 mm

Thickness of 5 papers = 0.016×5 = 0.08 mm

Total thickness of a stack = 100+0.08 = 100.08 mm

$= 100.08 \times 10^2/10^2$ mm

$= 1.0008 \times 10^2$ mm