EXERCISE: 7.1 (PAGE NO: 118)

1. In quadrilateral ACBD, AC = AD and AB bisect ∠A (see Fig. 7.16). Show that ΔABC≅ ΔABD. What can you say about BC and BD?

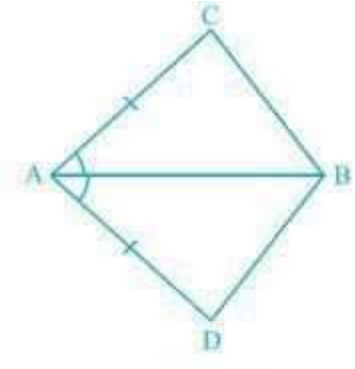

Fig. 7.16

Solution:

It is given that AC and AD are equal i.e. AC = AD and the line segment AB bisects ∠A. We will have to now prove that the two triangles ABC and ABD are similar i.e. **ΔABC ≅ ΔABD Proof:**

Consider the triangles ΔABC and ΔABD,

(i) AC = AD (It is given in the question)

(ii) AB = AB (Common)

(iii) ∠CAB = ∠DAB (Since AB is the bisector of angle A)

So, by **SAS congruency criterion**, ΔABC ≅ ΔABD.

For the 2nd part of the question, BC and BD are of equal lengths by the rule of C.P.C.T.

2. ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA (see Fig. 7.17). Prove that

(i) ΔABD ≅ ΔBAC

(ii) BD = AC

(iii) ∠ABD = ∠BAC.

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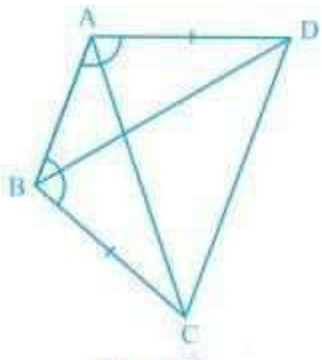

Fig. 7.17

Solution:

The given parameters from the questions are ∠DAB = ∠CBA and AD = BC.

(i) ΔABD and ΔBAC are similar by SAS congruency as

AB = BA (It is the common arm)

∠DAB = ∠CBA and AD = BC (These are given in the question)

So, triangles ABD and BAC are similar i.e. ΔABD ≅ ΔBAC. (Hence proved).

(ii) It is now known that ΔABD ≅ ΔBAC so, BD = AC (by the rule of CPCT).

(iii) Since ΔABD ≅ ΔBAC so,

Angles ∠ABD = ∠BAC (by the rule of CPCT).

3. AD and BC are equal perpendiculars to a line segment AB (see Fig. 7.18). Show that CD bisects AB.

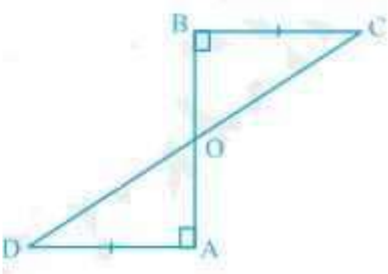

Fig. 7.18

Solution:

It is given that AD and BC are two equal perpendiculars to AB.

We will have to prove that **CD is the bisector of AB**

Now,

Triangles ΔAOD and ΔBOC are similar by AAS congruency since:

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(i) ∠A = ∠B (They are perpendiculars)

(ii) AD = BC (As given in the question)

(iii) ∠AOD = ∠BOC (They are vertically opposite angles)

∴ ΔAOD ≅ ΔBOC.

So, AO = OB (by the rule of CPCT).

Thus, CD bisects AB (Hence proved).

4. *l* and *m* are two parallel lines intersected by another pair of parallel lines p and q (see Fig. 7.19). Show that ΔABC ≅ ΔCDA.

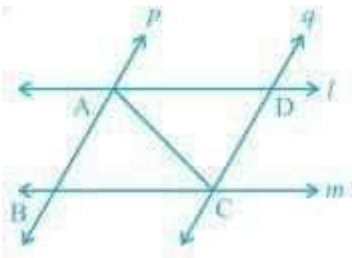

Fig. 7.19

Solution:

It is given that p || q and l || m **To**

prove:

Triangles ABC and CDA are similar i.e. ΔABC ≅ ΔCDA **Proof**:

Consider the ΔABC and ΔCDA,

(i) ∠BCA = ∠DAC and ∠BAC = ∠DCA Since they are alternate interior angles

(ii) AC = CA as it is the common arm

So, by **ASA congruency criterion,** ΔABC ≅ ΔCDA.

5. Line l is the bisector of an angle ∠A and B is any point on *l*. BP and BQ are perpendiculars from B to the arms of ∠A (see Fig. 7.20). Show that:

(i) ΔAPB ≅ ΔAQB

(ii) BP = BQ or B is equidistant from the arms of ∠A.

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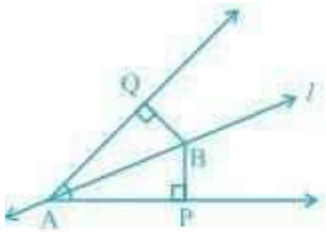

Fig. 7.20

Solution:

It is given that the line "*l*" is the bisector of angle ∠A and the line segments BP and BQ are perpendiculars drawn from *l*.

(i) ΔAPB and ΔAQB are similar by AAS congruency because:

∠P = ∠Q (They are the two right angles)

AB = AB (It is the common arm)

∠BAP = ∠BAQ (As line *l* is the bisector of angle A)

So, ΔAPB ≅ ΔAQB.

(ii) By the rule of CPCT, BP = BQ. So, it can be said the point B is equidistant from the arms of ∠A.

6. In Fig. 7.21, AC = AE, AB = AD and ∠BAD = ∠EAC. Show that BC = DE.

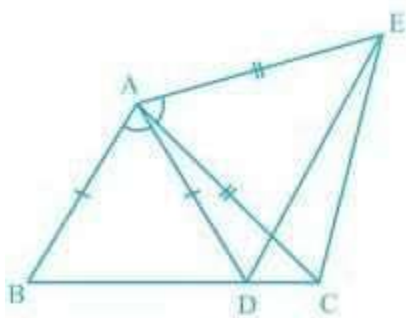

Fig. 7.21

Solution:

It is given in the question that AB = AD, AC = AE, and ∠BAD = ∠EAC **To prove:**

The line segment BC and DE are similar i.e. BC = DE **Proof:**

We know that ∠BAD = ∠EAC

Now, by adding ∠DAC on both sides we get,

∠BAD + ∠DAC = ∠EAC +∠DAC

This implies, ∠BAC = ∠EAD

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Now, ΔABC and ΔADE are similar by SAS congruency since:

(i) AC = AE (As given in the question)

(ii) ∠BAC = ∠EAD

(iii) AB = AD (It is also given in the question)

∴ Triangles ABC and ADE are similar i.e. ΔABC ≅ ΔADE.

So, by the rule of CPCT, it can be said that BC = DE.

7. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that ∠BAD = ∠ABE and ∠EPA = ∠DPB (see Fig. 7.22). Show that

(i) ΔDAP ≅ ΔEBP

(ii) AD = BE

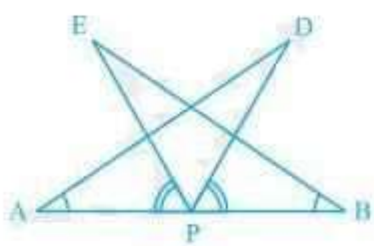

Fig. 7.22

Solutions:

In the question, it is given that P is the mid-point of line segment AB. Also, ∠BAD = ∠ABE and ∠EPA = ∠DPB

(i) It is given that ∠EPA = ∠DPB

Now, add ∠DPE on both sides,

∠EPA +∠DPE = ∠DPB+∠DPE

This implies that angles DPA and EPB are equal i.e. ∠DPA = ∠EPB Now,

consider the triangles DAP and EBP.

∠DPA = ∠EPB

AP = BP (Since P is the mid-point of the line segment AB)

∠BAD = ∠ABE (As given in the question)

So, by **ASA congruency**, ΔDAP ≅ ΔEBP.

(ii) By the rule of CPCT, AD = BE.

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8. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see Fig. 7.23). Show that:

(i) ΔAMC ≅ ΔBMD

(ii) ∠DBC is a right angle.

(iii) ΔDBC ≅ ΔACB

(iv) CM = ½ AB

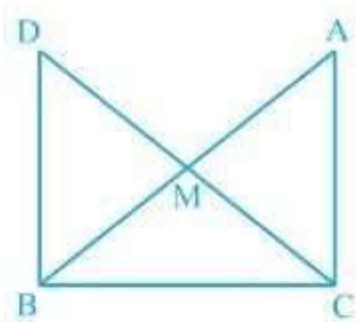

Fig. 7.23

Solution:

It is given that M is the mid-point of the line segment AB, ∠C = 90°, and DM = CM

(i) Consider the triangles ΔAMC and ΔBMD:

AM = BM (Since M is the mid-point)

CM = DM (Given in the question)

∠CMA = ∠DMB (They are vertically opposite angles) So,

by **SAS congruency criterion**, ΔAMC ≅ ΔBMD.

(ii) ∠ACM = ∠BDM (by CPCT)

∴ AC || BD as alternate interior angles are equal.

Now, ∠ACB +∠DBC = 180° (Since they are co-interiors angles)

⇒ 90° +∠B = 180°

∴ ∠DBC = 90°

(iii) In ΔDBC and ΔACB,

BC = CB (Common side)

∠ACB = ∠DBC (They are right angles)

DB = AC (by CPCT)

So, ΔDBC ≅ ΔACB by **SAS congruency**.

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(iv) DC = AB (Since ΔDBC ≅ ΔACB)

⇒ DM = CM = AM = BM (Since M the is mid-point)

So, DM + CM = BM+AM

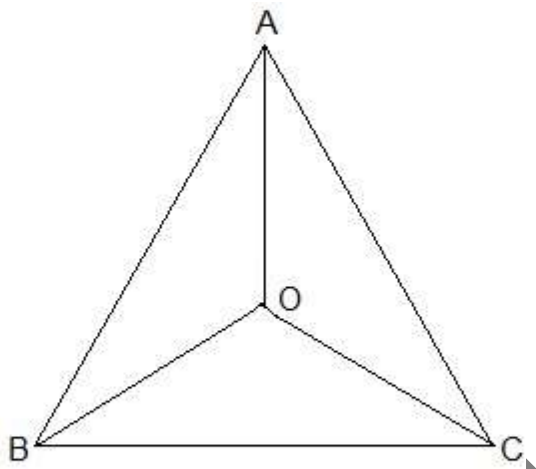
Hence, CM + CM = AB

⇒ CM = (½) AB

EXERCISE: 7.2 (PAGE NO: 123)

1. In an isosceles triangle ABC, with AB = AC, the bisectors of ∠B and ∠C intersect each other at O. Join A to O. Show that:

(i) OB = OC (ii) AO bisects ∠A



Solution:

Given:

AB = AC and

the bisectors of ∠B and ∠C intersect each other at O

(i) Since ABC is an isosceles with AB = AC,

∠B = ∠C

½ ∠B = ½ ∠C

⇒ ∠OBC = ∠OCB (Angle bisectors)

∴ OB = OC (Side opposite to the equal angles are equal.)

(ii) In ΔAOB and ΔAOC,

AB = AC (Given in the question)

AO = AO (Common arm)

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OB = OC (As Proved Already)

So, ΔAOB ≅ ΔAOC by SSS congruence condition.

BAO = CAO (by CPCT) Thus,

AO bisects ∠A.

2. In ΔABC, AD is the perpendicular bisector of BC (see Fig. 7.30). Show that ΔABC is an isosceles triangle in which AB = AC.

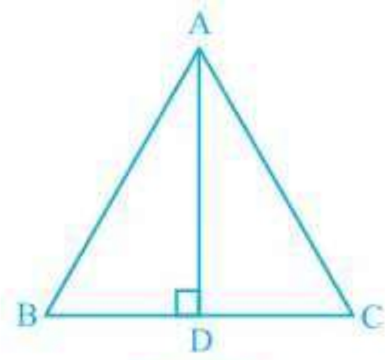

Fig. 7.30

Solution:

It is given that AD is the perpendicular bisector of BC **To**

prove:

AB = AC **Proof:**

In ΔADB and ΔADC,

AD = AD (It is the Common arm)

∠ADB = ∠ADC

BD = CD (Since AD is the perpendicular bisector)

So, ΔADB ≅ ΔADC by **SAS congruency criterion**.

Thus,

AB = AC (by CPCT)

3. ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (see Fig. 7.31). Show that these altitudes are equal.

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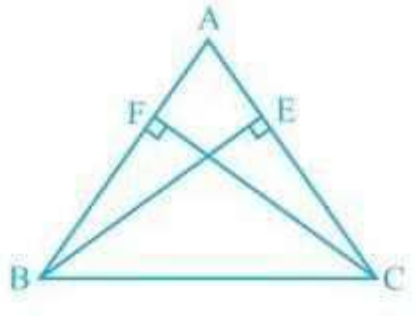

Fig. 7.31

Solution:

Given:

(i) BE and CF are altitudes.

(ii) AC = AB **To prove:**

BE = CF **Proof:**

Triangles ΔAEB and ΔAFC are similar by AAS congruency since

∠A = ∠A (It is the common arm)

∠AEB = ∠AFC (They are right angles)

AB = AC (Given in the question)

∴ ΔAEB ≅ ΔAFC and so, BE = CF (by CPCT).

4. ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (see Fig. 7.32). Show that

(i) ΔABE ≅ ΔACF

(ii) AB = AC, i.e., ABC is an isosceles triangle.

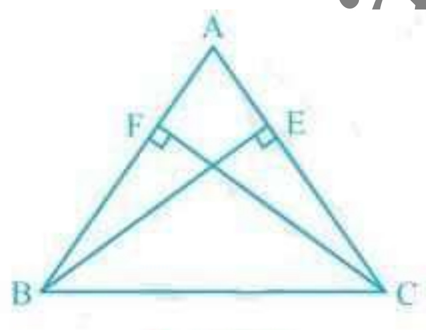

Fig. 7.32

Solution:

It is given that BE = CF

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(i) In ΔABE and ΔACF,

∠A = ∠A (It is the common angle)

∠AEB = ∠AFC (They are right angles)

BE = CF (Given in the question)

∴ ΔABE ≅ ΔACF by **AAS congruency condition**.

(ii) AB = AC by CPCT and so, ABC is an isosceles triangle.

5. ABC and DBC are two isosceles triangles on the same base BC (see Fig. 7.33). Show that ∠ABD = ∠ACD.

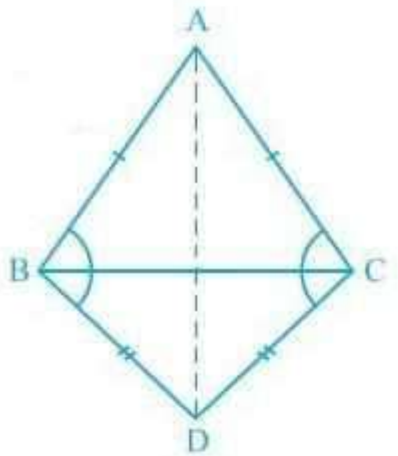

Fig. 7.33

Solution:

In the question, it is given that ABC and DBC are two isosceles triangles.

We will have to show that ∠ABD = ∠ACD **Proof:**

Triangles ΔABD and ΔACD are similar by $SS congruency since

AD = AD (It is the common arm)

AB = AC (Since ABC is an isosceles triangle)

BD = CD (Since BCD is an isosceles triangle)

So, ΔABD ≅ ΔACD.

∴ ∠ABD = ∠ACD by CPCT.

6. ΔABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB (see Fig. 7.34). Show that ∠BCD is a right angle.

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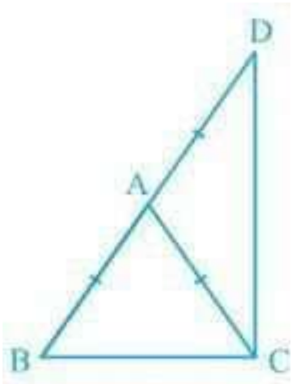

Fig. 7.34

Solution:

It is given that AB = AC and AD = AB We will have to now prove ∠BCD is a right angle.

Proof:

Consider ΔABC,

AB = AC (It is given in the question)

Also, ∠ACB = ∠ABC (They are angles opposite to the equal sides and so, they are equal)

Now, consider ΔACD,

AD = AB

Also, ∠ADC = ∠ACD (They are angles opposite to the equal sides and so, they are equal) Now,

In ΔABC,

∠CAB + ∠ACB + ∠ABC = 180°

So, ∠CAB + 2∠ACB = 180°

⇒ ∠CAB = 180° – 2∠ACB — (i)

Similarly, in ΔADC,

∠CAD = 180° – 2∠ACD — (ii)

also,

∠CAB + ∠CAD = 180° (BD is a straight line.)

Adding (i) and (ii) we get,

∠CAB + ∠CAD = 180° – 2∠ACB+180° – 2∠ACD

⇒ 180° = 360° – 2∠ACB-2∠ACD

⇒ 2(∠ACB+∠ACD) = 180°

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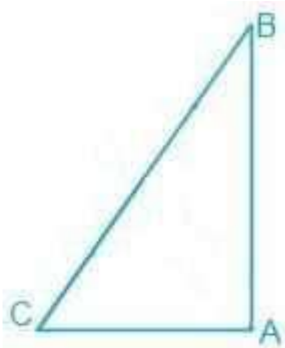
⇒ ∠BCD = 90°

7. ABC is a right-angled triangle in which ∠A = 90° and AB = AC. Find ∠B and ∠C.

Solution:



In the question, it is given that

∠A = 90° and AB = AC

AB = AC

⇒ ∠B = ∠C (They are angles opposite to the equal sides and so, they are equal)

Now,

∠A+∠B+∠C = 180° (Since the sum of the interior angles of the triangle)

∴ 90° + 2∠B = 180°

⇒ 2∠B = 90°

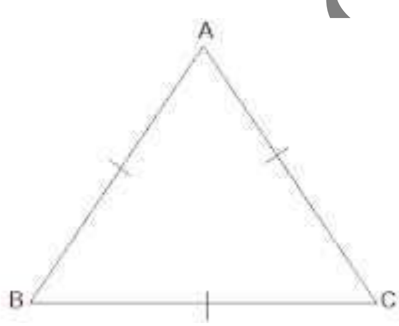
⇒ ∠B = 45°

So, ∠B = ∠C = 45°

8. Show that the angles of an equilateral triangle are 60° each.

Solution:

Let ABC be an equilateral triangle as shown below:



Here, BC = AC = AB (Since the length of all sides is same)

⇒ ∠A = ∠B =∠C (Sides opposite to the equal angles are equal.)

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Also, we know that

∠A+∠B+∠C = 180°

⇒ 3∠A = 180°

⇒ ∠A = 60°

∴ ∠A = ∠B = ∠C = 60°

So, the angles of an equilateral triangle are always 60° each.

EXERCISE: 7.3 (PAGE NO: 128)

1. ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (see Fig. 7.39). If AD is extended to intersect BC at P, show that

(i) ΔABD ≅ ΔACD

(ii) ΔABP ≅ ΔACP

(iii) AP bisects ∠A as well as ∠D.

(iv) AP is the perpendicular bisector of BC.

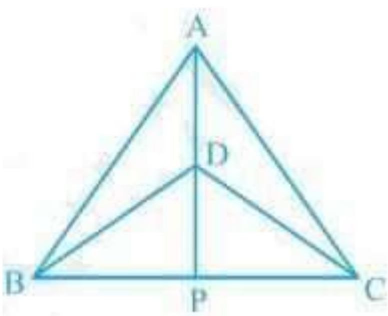

Fig. 7.39

Solution:

In the above question, it is given that ΔABC and ΔDBC are two isosceles triangles.

(i) ΔABD and ΔACD are similar by SSS congruency because:

AD = AD (It is the common arm)

AB = AC (Since ΔABC is isosceles)

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BD = CD (Since ΔDBC is isosceles) ∴

ΔABD ≅ ΔACD.

(ii) ΔABP and ΔACP are similar as:

AP = AP (It is the common side)

∠PAB = ∠PAC (by CPCT since ΔABD ≅ ΔACD)

AB = AC (Since ΔABC is isosceles)

So, ΔABP ≅ ΔACP by SAS congruency condition.

(iii) ∠PAB = ∠PAC by CPCT as ΔABD ≅ ΔACD.

AP bisects ∠A. — (i)

Also, ΔBPD and ΔCPD are similar by SSS congruency as

PD = PD (It is the common side)

BD = CD (Since ΔDBC is isosceles.)

BP = CP (by CPCT as ΔABP ≅ ΔACP) So,

ΔBPD ≅ ΔCPD.

Thus, ∠BDP = ∠CDP by CPCT. — (ii)

Now by comparing (i) and (ii) it can be said that AP bisects ∠A as well as ∠D.

(iv) ∠BPD = ∠CPD (by CPCT as ΔBPD ΔCPD) and BP = CP — (i) also,

∠BPD +∠CPD = 180° (Since BC is a straight line.)

⇒ 2∠BPD = 180°

⇒ ∠BPD = 90° —(ii)

Now, from equations (i) and (ii), it can be said that AP

is the perpendicular bisector of BC.

2. AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that (i) AD bisects BC (ii) AD bisects ∠A.

Solution:

It is given that AD is an altitude and AB = AC. The diagram is as follows:

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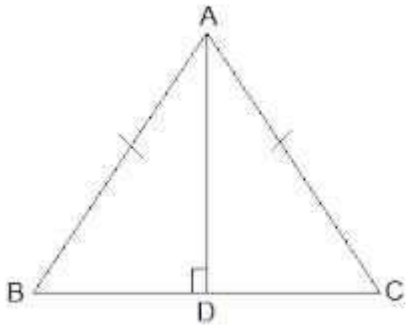

(i) In ΔABD and ΔACD,

∠ADB = ∠ADC = 90°

AB = AC (It is given in the question)

AD = AD (Common arm)

∴ ΔABD ≅ ΔACD by RHS congruence condition.

Now, by the rule of CPCT, BD

= CD.

So, AD bisects BC

(ii) Again, by the rule of CPCT, ∠BAD = ∠CAD Hence, AD bisects ∠A.

3. Two sides AB and BC and median AM of one triangle ABC are respectively equal to sides PQ and QR and median PN of ΔPQR (see Fig. 7.40). Show that:

(i) ΔABM ≅ ΔPQN

(ii) ΔABC ≅ ΔPQR

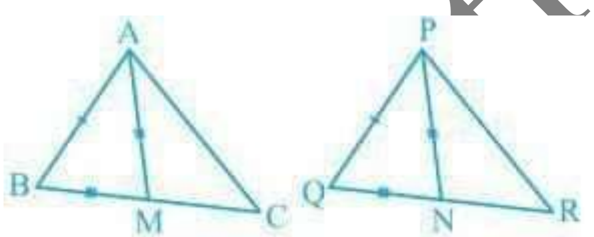

Fig. 7.40

Solution:

Given parameters are:

AB = PQ,

BC = QR and

AM = PN

(i) ½ BC = BM and ½ QR = QN (Since AM and PN are medians)

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Also, BC = QR

So, ½ BC = ½ QR

⇒ BM = QN

In ΔABM and ΔPQN,

AM = PN and AB = PQ (As given in the question)

BM = QN (Already proved)

∴ ΔABM ≅ ΔPQN by SSS congruency.

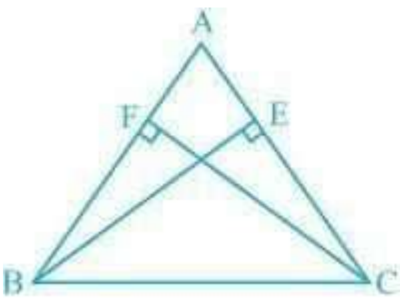
(ii) In ΔABC and ΔPQR,

AB = PQ and BC = QR (As given in the question)

∠ABC = ∠PQR (by CPCT)

So, ΔABC ≅ ΔPQR by SAS congruency.

4. BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.



Solution:

It is known that BE and CF are two equal altitudes.

Now, in ΔBEC and ΔCFB,

∠BEC = ∠CFB = 90° (Same Altitudes)

BC = CB (Common side)

BE = CF (Common side)

So, ΔBEC ≅ ΔCFB by RHS congruence criterion.

Also, ∠C = ∠B (by CPCT)

Therefore, AB = AC as sides opposite to the equal angles is always equal.

5. ABC is an isosceles triangle with AB = AC. Draw AP ⊥ BC to show that ∠B = ∠C.

Solution:

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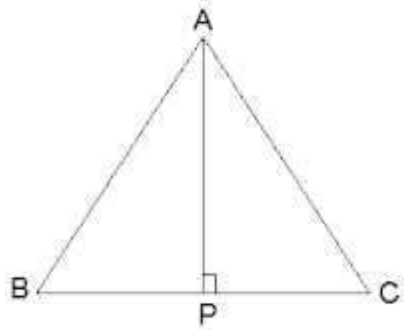


In the question, it is given that AB = AC

Now, ΔABP and ΔACP are similar by RHS congruency as

$\angle APB = \angle APC = 90^\circ$ (AP is altitude)

AB = AC (Given in the question)

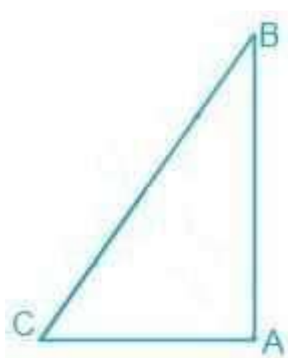
AP = AP (Common side)

So, ΔABP ≅ ΔACP.

$\therefore \angle B = \angle C$ (by CPCT)

EXERCISE: 7.4 (PAGE NO: 132)

1. Show that in a right-angled triangle, the hypotenuse is the longest side.



Solution:

It is known that ABC is a triangle right angled at B.

We know that,

$\angle A + \angle B + \angle C = 180^\circ$

Now, if $\angle B + \angle C = 90^\circ$ then $\angle A$ has to be 90°.

Since A is the largest angle of the triangle, the side opposite to it must be the largest.

So, AB is the hypotenuse which will be the largest side of the above right-angled triangle i.e. ΔABC.

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2. In Fig. 7.48, sides AB and AC of ΔABC are extended to points P and Q respectively. Also, ∠PBC < ∠QCB. Show that AC > AB.

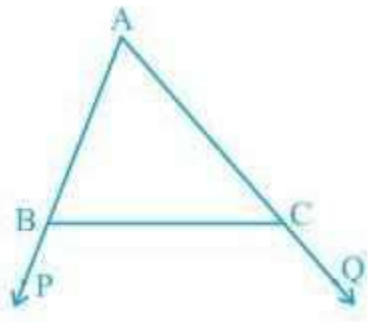

Fig. 7.48

Solution:

It is given that ∠PBC < ∠QCB

We know that ∠ABC + ∠PBC = 180°

So, ∠ABC = 180°-∠PBC

Also,

∠ACB +∠QCB = 180°

Therefore ∠ACB = 180° -∠QCB

Now, since ∠PBC < ∠QCB,

∴ ∠ABC > ∠ACB

Hence, AC > AB as sides opposite to the larger angle is always larger.

3. In Fig. 7.49, ∠B < ∠A and ∠C < ∠D. Show that AD < BC.

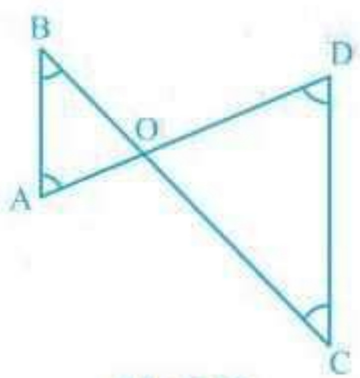

Fig. 7.49

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Solution:

In the question, it is mentioned that angles B and angle C is smaller than angles A and D respectively i.e. ∠B < ∠A and ∠C < ∠D.

Now,

Since the side opposite to the smaller angle is always smaller

AO < BO — (i)

And OD < OC —(ii)

By adding equation (i) and equation (ii) we get

AO+OD < BO + OC So,

AD < BC

4. AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (see Fig. 7.50).

Show that ∠A > ∠C and ∠B > ∠D.

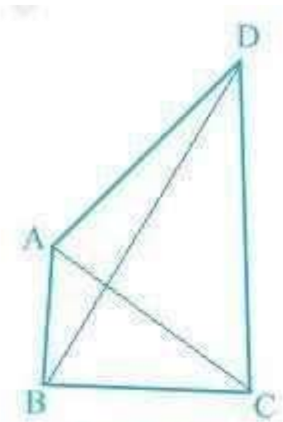

Fig. 7.50

Solution:

In ΔABD, we see that

AB < AD < BD

So, ∠ADB < ∠ABD — (i) (Since angle opposite to longer side is always larger)

Now, in ΔBCD,

BC < DC < BD

Hence, it can be concluded that

∠BDC < ∠CBD — (ii)

Now, by adding equation (i) and equation (ii) we get,

∠ADB + ∠BDC < ∠ABD + ∠CBD

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∠ADC < ∠ABC

∠B > ∠D

Similarly, In triangle ABC,

∠ACB < ∠BAC — (iii) (Since the angle opposite to the longer side is always larger)

Now, In ΔADC,

∠DCA < ∠DAC — (iv)

By adding equation (iii) and equation (iv) we get,

∠ACB + ∠DCA < ∠BAC+∠DAC

⇒ ∠BCD < ∠BAD

∴ ∠A > ∠C

5. In Fig 7.51, PR > PQ and PS bisect ∠QPR. Prove that ∠PSR > ∠PSQ.

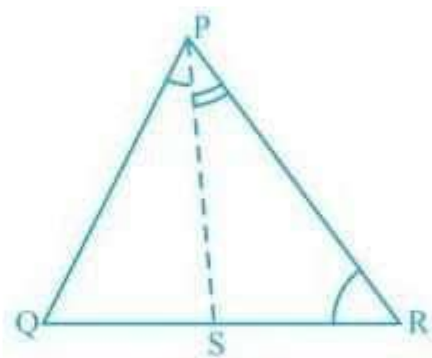

Fig. 7.51

Solution:

It is given that PR > PQ and PS bisects ∠QPR

Now we will have to prove that angle PSR is smaller than PSQ i.e. ∠PSR > ∠PSQ **Proof:**

∠QPS = ∠RPS — (ii) (As PS bisects ∠QPR)

∠PQR > ∠PRQ — (i) (Since PR > PQ as angle opposite to the larger side is always larger)

∠PSR = ∠PQR + ∠QPS — (iii) (Since the exterior angle of a triangle equals to the sum of opposite interior angles)

∠PSQ = ∠PRQ + ∠RPS — (iv) (As the exterior angle of a triangle equals to the sum of opposite interior angles) By adding (i) and (ii)

∠PQR +∠QPS > ∠PRQ +∠RPS

Thus, from (i), (ii), (iii) and (iv), we get

∠PSR > ∠PSQ

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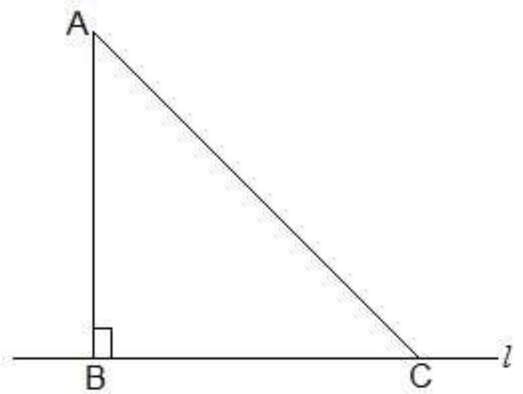
6. Show that of all line segments drawn from a given point not on it, the perpendicular line segment is the shortest. Solution:

First, let "*l*" be a line segment and "B" be a point lying on it. A line AB perpendicular to *l* is now drawn. Also, let C be any other point on *l*. The diagram will be as follows:



To prove:

AB < AC **Proof:**

In ΔABC, ∠B = 90°

Now, we know that

∠A+∠B+∠C = 180°

∴ ∠A +∠C = 90°

Hence, ∠C must be an acute angle which implies ∠C < ∠B

So, AB < AC (As the side opposite to the larger angle is always larger)

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